10th Class 2018

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Write the quadratic equation $\frac{x}{x+1} + \frac{x+1}{x} = 6$.

Ans Given,

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$

$$\frac{x(x) + (x+1)(x+1)}{(x+1)(x)} = 6$$

$$\frac{x^2 + (x+1)^2}{x^2 + x} = 6$$

$$x^2 + x^2 + 1 + 2x = 6(x^2 + x)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$\Rightarrow$ $\boxed{4x^2 + 4x - 1 = 0}$ Ans

(ii) Write the standard quadratic equation and also write quadratic formula to solve it.

Ans The standard quadratic equation is:

$$ax^2 + bx + c = 0$$

Quadratic formula to solve it

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(iii) Find the sum and product of the roots of the equation $2px^2 + 3qx - 4r = 0$ without solving.

Ans Given, $2px^2 + 3qx - 4r = 0$

Here, $a = 2p$, $b = 3q$, $c = -4r$

Sum of the roots $= \frac{-b}{a}$

$$= \frac{-3q}{2p}$$

Product of the roots $= \frac{c}{a}$

$$= \frac{-4r}{2p}$$

$$= \frac{-2r}{p}$$

(iv) Form a quadratic equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$.

Ans Here, $\alpha = 3 + \sqrt{2}$

$\beta = 3 - \sqrt{2}$

Sum of the roots $= \alpha + \beta$

$$= (3 + \sqrt{2}) + (3 - \sqrt{2})$$

$$= 3 + \sqrt{2} + 3 - \sqrt{2}$$

$$= 6$$

Product of the roots $= \alpha\beta$

$$= (3 + \sqrt{2})(3 - \sqrt{2})$$

$$= (3)^2 - (\sqrt{2})^2$$

$$= 9 - 2$$

$$= 7$$

To find quadratic equation,

$$x^2 - Sx + P = 0$$

By putting the values, we get the required quadratic equations:

$$x^2 - 6x + 7 = 0$$

(v) Evaluate: $(1 - 3\omega - 2\omega^2)^5$

Ans $(1 - 3\omega - 3\omega^2)^5 = [1 - 3(\omega + \omega^2)]^5$

$$= [1 - 3(-1)]^5 \qquad \because \omega + \omega^2 = -1$$

$$= (1 + 3)^5$$

$$= 4^5$$

$$= 1024$$

(vi) **Define synthetic division.**

Ans Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In fact, synthetic division is simply a short-cut of long division method.

(vii) **Find p, if 12, p and 3 are in continued proportion.**

Ans Since 12, p and 3 are in continued proportion.

$\therefore \quad 12 : p :: p : 3$

i.e., $(p)(p) = (12)(3)$

$p^2 = 36$

Thus, $p = \pm 6$

(viii) **Find the ratio x : y, if 3(4x – 5y) = 2x – 7y.**

Ans Given,

$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = -7y + 15y$$
$$10x = 8y$$
$$\Rightarrow \quad \frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5}$$
$$x : y = 4 : 5 \quad \text{Ans.}$$

(ix) **Find a fourth proportional to 5, 8, 15.**

Ans Let a fourth proportional is x;

So,

$$5 : 8 :: 15 : x$$

Product of Extremes = Product of Means

$$5(x) = 8 \times 15$$
$$x = \frac{8 \times 15}{5}$$
$$x = 8 \times 3$$
$$x = 24$$

So, the fourth proportional is $x = 24$.

3. Write short answers to any SIX (6) questions: 12

(i) What is an improper fraction?

Ans A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction, if degree of the polynomial $N(x)$ is greater or equal to the degree of the polynomial $D(x)$.

For example:

$$\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}.$$

(ii) Find partial fraction of $\frac{3}{(x+1)(x-1)}$.

Ans Let,

$$\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \text{(i)}$$

By multiplying $(x + 1)(x - 1)$, we get

$$\frac{3}{(x+1)(x-1)}(x+1)(x-1) = \frac{A}{(x+1)}(x+1)(x-1) + \frac{B}{(x-1)}(x+1)(x-1)$$

$$3 = A(x-1) + B(x+1) \quad \text{(ii)}$$

As, $x - 1 = 0$

$x = 1$

Put $x = 1$ in (ii),

$3 = A(1-1) + B(1+1)$

$3 = A(0) + 2B$

$3 = 2B$

$\frac{3}{2} = B$

$\Rightarrow B = \frac{3}{2}$

And $x + 1 = 0$

$x = -1$

Put $x = -1$ in (ii),

$3 = A(-1-1) + B(-1+1)$

$$3 = A(-2) + B(0)$$
$$3 = -2A$$
$$\frac{3}{-2} = A$$
$$\Rightarrow \quad A = \frac{-3}{2}$$

By putting the values of A and B in (i),

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)}$$

(iii) If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9}, then find $Y \cap X$.

Ans Given, X = {1, 4, 7, 9} , Y = {2, 4, 5, 9}

$$Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$$
$$= \{4, 9\}$$

(iv) If X = {1, 3, 5, 7, ----, 9} Y = {0, 2, 4, 6, 8, ----, 20} and Z = {2, 3, 5, 7, 11, 13, 17, 19, 23}, then find $(X \cap Y) \cap Z$.

Ans $(X \cap Y) = (\{1, 3, 5, 7, \ldots, 9\} \cap \{0, 2, 4, 6, 8, \ldots, 20\}) \cap Z$

$$= \{\ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{\ \}$$

(v) Find a and b if (2a + 5, 3) = (7, b − 4).

Ans Given (2a + 5, 3) = (7, b − 4)

By comparing, we get

Firstly,

$$2a + 5 = 7$$
$$2a = 7 - 5$$
$$2a = 2$$
$$a = \frac{2}{2}$$
$$\boxed{a = 1}$$

And

$$3 = b - 4$$
$$3 + 4 = b$$
$$7 = b$$
$$\Rightarrow \quad \boxed{b = 7}$$

(vi) **Define an onto function.**

Ans A function $f : A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A *i.e.*, Range of f = B.

(vii) **Define a frequency distribution.**

Ans A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observations falling in each group corresponds to the respective group.

(viii) **Find arithmetic mean by direct method:**

200, 225, 350, 375, 270, 320, 290.

Ans The arithmetic Mean:

$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{200 + 225 + 350 + 375 + 270 + 320 + 290}{7}$$

$$= \frac{2030}{7}$$

$$\boxed{\bar{X} = 290}$$

(ix) **For the following data, find the harmonic mean:**

x	12	5	8	4

Ans

X	$\frac{1}{X}$
12	0.0833
5	0.2
8	0.125
4	0.25
	0.6583

$$H.M = \frac{n}{\Sigma(\frac{1}{x})} = \frac{4}{0.6583}$$

$\boxed{H.M = 6.0763}$

4. Write short answers to any SIX (6) questions: (12

(i) Verify the identity: $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$.

Ans Given,

$$(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$$

$$\begin{aligned} L.H.S &= (1 - \sin\theta)(1 + \sin\theta) \\ &= (1)^2 - (\sin\theta)^2 \\ &= 1 - \sin^2\theta \\ &= (\sin^2\theta + \cos^2\theta) - \sin^2\theta \\ &= \sin^2\theta + \cos^2\theta - \sin^2\theta \\ &= \cos^2\theta \\ &= R.H.S \end{aligned}$$

(ii) How many minutes are there in two right angles?

Ans As we know that:

1 degree = 60 minutes

Two right angles have 180 degrees

Thus

Two right angles = 180 × 60 minutes

= 10,800 minutes

(iii) Find 'r', when $l = 52$ cm, $\theta = 45^\circ$.

Ans As we know that,

$$\theta = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

$$\begin{aligned} r &= \frac{l}{\theta} \\ &= l \div \theta \\ &= 52 \div \frac{\pi}{4} \\ &= 52 \times \frac{4}{\pi} \\ &= 52 \times 1.273 \end{aligned}$$

$\boxed{r = 66.21}$

(iv) **What is meant by zero dimension?**

Ans The projection of a finite line on another line is the portion of the latter intercepted between the projection of ends of the given finite line. However, projection of a vertical line on another line is the join of these two interesting lines, which is of zero dimension.

(v) **Define circumference.**

Ans The length of the boundary of the circle is called the circumference.

(vi) **Define secant.**

Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vii) **Define chord of a circle.**

Ans The join of any two points on the circumference of the circle is called its chord.

(viii) **Define cyclic quadrilateral.**

Ans A quadrilateral is called cyclic when a circle can be drawn through its four vertices.

(ix) **Define an arc.**

Ans A part of circumference of a circle is called an arc.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: **(4)**

$$5x^{1/2} = 7x^{1/4} - 2$$

Ans Let

$$x^{1/4} = y$$

$$(x^{1/4})^2 = (y)^2$$

$$x^{1/2} = y^2$$

By putting the values in the given expression, we get

$$5(y^2) = 7(y) - 2$$

$$5y^2 = 7y - 2$$

$$5y^2 - 7y + 2 = 0$$

$$5y^2 - 5y - 2y + 2 = 0$$

$$5y(y - 1) - 2(y - 1) = 0$$

$$(5y-2)(y-1)=0$$

$5y-2=0$; $y-1=0$

$5y=2$; $\boxed{y=1}$

$$\boxed{y=\frac{2}{5}}$$

As $y = x^{1/4}$

So, $x^{1/4} = 1$; $x^{1/4} = \frac{2}{5}$

Taking square on both sides ; Taking square on both sides

$(x^{1/4})^2 = (1)^2$; $(x^{1/4})^2 = \left(\frac{2}{5}\right)^2$

$x^{1/2} = 1$; $x^{1/2} = \frac{4}{25}$

Again taking square on both sides ; Again taking square on both sides

$(x^{1/2})^2 = (1)^2$; $(x^{1/2})^2 = \left(\frac{4}{25}\right)^2$

$x = 1$; $x = \frac{16}{625}$

$$S.S = \left\{1, \frac{16}{625}\right\}$$

(b) Find the value of h using synthetic division, if 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$. (4)

Ans As 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$.

$$P(x) = 2x^3 - 3hx^2 + 9$$
$$= 2x^3 - 3hx^2 + 0x + 9$$

And $a = 3$

So,

3	2	−3h	0	9
	↓	6	−9h + 18	−27h + 54
	2	−3h + 6	−9h + 18	−27h + 63

As 3 is the zero of polynomial, then R = 0

$$R = -27h + 63 = 0$$
$$-27h = -63$$
$$h = \frac{-63}{-27}$$
$$h = \frac{7}{3}$$

Q.6.(a) Using componendo-dividendo theorem, solve the equation $\frac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}} = \frac{4}{3}$. (4)

Ans Given equation is $\frac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}} = \frac{4}{3}$

By using componendo-dividendo theorem,

$$\frac{\sqrt{x+3}+\sqrt{x-3}+\sqrt{x+3}-\sqrt{x-3}}{\sqrt{x+3}+\sqrt{x-3}-\sqrt{x+3}+\sqrt{x-3}} = \frac{4+3}{4-3}$$
$$\frac{2\sqrt{x+3}}{2\sqrt{x-3}} = \frac{7}{1}$$
$$\sqrt{\frac{x+3}{x-3}} = 7$$

Squaring both sides, we get

$$\frac{x+3}{x-3} = 49$$

$$x + 3 = 49(x - 3)$$
$$x + 3 = 49x - 147$$
$$x - 49x = -147 - 3$$
$$-48x = -150$$
$$48x = 150$$
$$x = \frac{150}{48}$$
$$x = \frac{25}{8}$$

(b) Resolve into partial fractions: (4)

$$\frac{1}{(x^2-1)(x+1)}$$

Ans
$$\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)(x+1)}$$
$$= \frac{1}{(x+1)^2(x-1)}$$

So,
$$\frac{1}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} \quad \text{(i)}$$

By multiplying $(x+1)^2(x-1)$, we get
$$1 = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \quad \text{(ii)}$$
$$1 = A(x^2-1) + B(x-1) + C(x^2+2x+1) \quad \text{(iii)}$$

Put $x+1=0$, *i.e.*, $x=-1$ in (ii),
$$1 = A((-1)^2-1) + B(-1-1) + C((-1)^2+2(-1)+1$$
$$1 = A(0) + B(-1-1) + C(0)$$
$$1 = B(-1-1)$$
$$1 = -2B$$
$$\boxed{B = \frac{-1}{2}}$$

Put $x-1=0$, *i.e.*, $x=1$ in (ii),
$$1 = C(1+1)^2$$
$$1 = C(2)^2$$
$$1 = 4C$$
$$\Rightarrow \quad \boxed{C = \frac{1}{4}}$$

By comparison the coefficients of x^2 in (iii),
$$0 = A + C$$
Put the value of C,
$$0 = A + \frac{1}{4}$$
$$\Rightarrow \quad \boxed{A = \frac{-1}{4}}$$

Put the values of A, B, C in (i),

$$\frac{1}{(x+1)^2(x-1)} = \frac{-1}{4(x+1)} - \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)}$$

So, $$\frac{1}{(x+1)^2(x-1)} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$

Q.7.(a) If U = {1, 2, 3, 4, - - - - , 10}, A = {1, 3, 5, 7, 9} and B = {1, 4, 7, 10}, then verify that B – A = B ∩ A′. (4)

Ans L.H.S = B – A

= {1, 4, 7, 10} – {1, 3, 5, 7, 9}

= {4, 10}

R.H.S = B ∩ A′

Now A′ = U – A

= {1, 2, 3, 4, ..., 10} – {1, 3, 5, 7, 9}

= {2, 4, 6, 8, 10}

R.H.S = B ∩ A′

= {1, 4, 7, 10)} ∩ {2, 4, 6, 8, 10}

= {4, 10}

So, L.H.S = R.H.S

(b) Calculate variance for the data: (4)

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Ans
$$\bar{X} = \frac{\Sigma X}{n}$$

$$= \frac{10+8+9+7+5+12+8+6+8+2}{10}$$

$$= \frac{75}{10}$$

$$\boxed{\bar{X} = 7.5}$$

X	$x - \bar{X}$	$(x - \bar{X})^2$
10	2.5	6.25
8	0.5	0.25
9	1.5	2.25
7	–0.5	0.25
5	–2.5	6.25
12	4.5	20.25

8	0.5	0.25
6	–1.5	2.25
8	0.5	0.25
2	–5.5	30.25
		68.5

$$\text{Variance (x)} = \frac{\Sigma(x - \bar{X})^2}{n}$$

$$= \frac{68.5}{10}$$

$$\boxed{\text{Var (x)} = 6.85}$$

Q.8.(a) Verify: $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$. (4)

Ans

$$\text{L.H.S} = (\tan\theta + \cot\theta)\tan\theta$$

$$= \tan\theta\tan\theta + \tan\theta\cot\theta$$

$$= \tan^2\theta + \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} + 1$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$

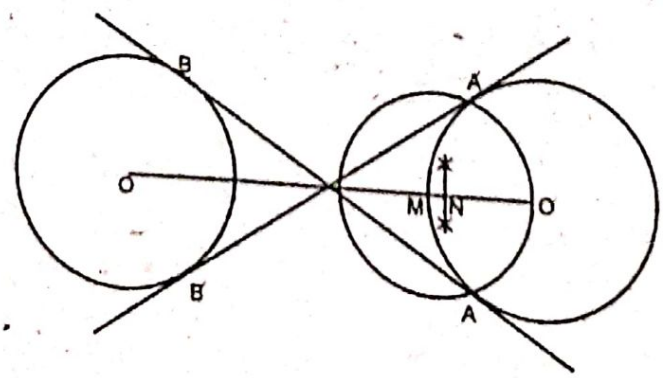
$$= \frac{1}{\cos^2\theta}$$

$$= \sec^2\theta$$

$$= \text{R.H.S}$$

(b) Draw two equal circles of each radius 2.4 cm. If the distance between their centres is 6 cm, then draw their transverse tangents. (4)

Ans



Steps of Construction:

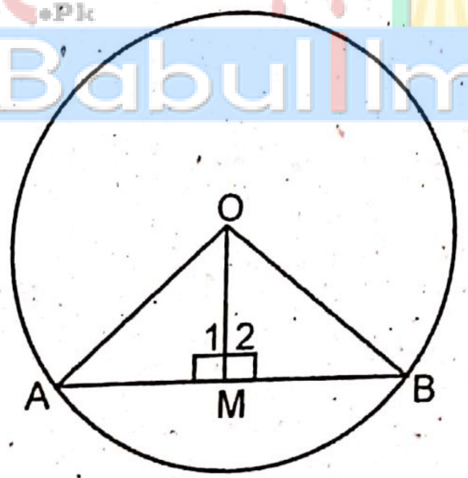
1. Draw a line segment $m\overline{OO'} = 6$ cm.
2. Draw two circles of 2.4 cm radius on O and O′.
3. Take M as the mid-point of $\overline{OO'}$ and N as the mid-point of $\overline{MO'}$.
4. Draw a circle with centre at N and a radius $\overline{NO'}$. This circle intersects the circle AA′.
5. Join A′ with M and produce towards M, it touch the second circle at B′.
6. Join A with M and produce towards M. AM produced touches the second circle at B.

So, A′B′ are the required tangents.

Q.9. Prove that a straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. (8)

Ans Given:

M is the mid-point of any chord $\overline{AB}$ of a circle which centre at O. Where chord $\overline{AB}$ is not the diameter of the circle.



To prove:

$\overline{OM} \perp$ the chord $\overline{AB}$.

Construction:

Join A and B with centre O. Write ∠1 & ∠2 as shown in the figure.

Proof:

Statement	Reasons
In $\Delta OAM \leftrightarrow \Delta OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\because \Delta OAM \cong \Delta OBM$	S.S.S ≅ S.S.S
$\Rightarrow m\angle 1 = m\angle 2$	
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ$	Adjacent supplementary angle
$m\angle 1 = m\angle 2 = 90^\circ$	
$\overline{OM} \perp \overline{AB}$	From (i) & (ii)

OR

Prove that the measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.

Ans For Answer see Paper 2017 (Group-I), Q.9.(OR).