10th Class 2019

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define reciprocal equation.

Ans An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

(ii) Solve by factorization: $5x^2 = 15x$

Ans Given: $5x^2 = 15x$

$$5x^2 - 15x = 0$$
$$5x(x-3) = 0$$
$$5x = 0 \quad ; \quad x - 3 = 0$$
$$x = 0 \quad ; \quad x = 3$$

So, the solution set = {0, 3}.

(iii) Find discriminant of the quadratic equation:

$$4x^2 - 7x - 2 = 0$$

Ans Here: $a = 4, b = -7, c = -2$

$$\text{Discriminant} = b^2 - 4ac$$
$$= (-7)^2 - 4(4)(-2)$$
$$= 49 + 32$$
$$= 81$$

(iv) Evaluate: $(9 + 4\omega + 4\omega^2)^3$

Ans Given: $(9 + 4\omega + 4\omega^2)^3$

$$= [9 + 4(\omega + \omega^2)]^3$$
$$= [9 + 4(-1)]^3 \quad \because \omega + \omega^2 = -1$$
$$= (9-4)^3$$
$$= 5^3 = 125$$

(v) Write the quadratic equation having roots 4, 9.

Ans As 4 and 9 are the roots of the required quadratic equation, so

Sum of roots: $S = 4 + 9 = 13$

Product of roots: $P = 4(9) = 36$

General quadratic equation, having roots, is

$$x^2 - Sx + P = 0 \qquad \text{(i)}$$

By putting the values in (i), we get the required quadratic equation, as:

$$x^2 - 13x + 36 = 0$$

(vi) Using synthetic division, divide $p(x) = x^4 - x^2 + 15$ by $x + 1$.

Ans

$$(x^4 - x^2 + 15) \div (x + 1)$$

As $x + 1 = x - (-1)$,

So, $a = -1$

Now, write the coefficients of dividend in a row and $a = -1$ on the left side.

	1	0	−1	0	15
−1	↓	−1	1	0	0
	1	−1	0	0	15

$\therefore$ Quotient $= Q(x) = x^3 - x^2 + 0.x + 0$

$Q(x) = x^3 - x^2$

and Remainder = 15

(vii) If $3(4x - 5y) = 2x - 7y$, find the ratio $x : y$.

Ans

$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = -7y + 15y$$
$$10x = 8y$$
$$\frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5}$$

By converting the above fraction into ratio, we get

$$x : y = 4 : 5$$

(viii) Find the fourth proportional to: 8, 7, 6.

Ans Let x be the fourth proportional, then

$$8 : 7 :: 6 : x$$

Product of extremes = Product of means

$$8(x) = 7(6)$$

$$x = \frac{42}{8}$$

Hence, Fourth Proportional: $\boxed{x = \frac{21}{4}}$

(ix) **Define joint variation.**

Ans A combination of direct and inverse variations of one or more than one variables forms joint variation.

3. Write short answers to any SIX (6) questions: (12)

(i) **Define fraction.**

Ans A fraction is an indicated quotient of two numbers or algebraic expressions.

(ii) **Define De-Morgan's laws.**

Ans For any two sets A and B, De-Morgan's laws are:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

(iii) **If A = {1, 3, 5, 7, 9} and B = {1, 4, 7, 10}, then find (A – B).**

Ans $A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

$= \{3, 5, 9\}$

(iv) **If A = {a, b} and B = {c, d}, then find A × B and B × A.**

Ans Given, $A = \{a, b\}$ and $B = \{c, d\}$

$A \times B = \{a, b\} \times \{c, d\}$

$= \{(a, c), (a, d), (b, c), (b, d)\}$

$B \times A = \{c, d\} \times \{a, b\}$

$= \{(c, a), (c, b), (d, a), (d, b)\}$

(v) **Find domain and the range of R = {(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)}.**

Ans From above function:

Dom R = {1, 2, 3, 4, 5}

Range R = {1, 3, 4}

(vi) **Define arithmetic mean and give an example.**

Ans **Arithmetic Mean:**

Arithmetic Mean (or simply called Mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. In symbols,

$$\bar{X} = \frac{\Sigma x}{n} \quad \text{(For ungrouped data)}$$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} \quad \text{(Grouped data)}$$

Example:

Marks of each student = 45, 60, 74, 58, 65, 63, 49

No. of values = n = 7

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{45 + 60 + 74 + 58 + 65 + 63 + 49}{7}$$

$$\bar{x} = \frac{414}{7}$$

$$\bar{x} = 59.14 \text{ marks}$$

(vii) Find range for the weights of students: 110, 109, 84, 89, 77, 104, 74, 97, 49, 59, 103, 62.

Ans Maximum value = X_m = 110

Minimum value = X_0 = 49

So, Range = $X_m - X_0$

= 110 – 49

= 61

(viii) On 5 terms test in mathematics, a student has made marks of 82, 93, 86, 92 and 79. Find the median for the marks.

Ans By arranging the marks in ascending order, the arranged data is:

79, 82, 86, 92, 93

Since number of observations is odd, *i.e.*, n = 5.

$$\text{Median} = \tilde{x} = \text{size of } \left(\frac{n+1}{2}\right)\text{th observation}$$

$$\tilde{x} = \text{size of } \left(\frac{5+1}{2}\right)\text{th observation}$$

$$\tilde{x} = \text{size of } 3^{rd} \text{ observation}$$

$$\tilde{x} = 86$$

(ix) **For the following data, find the harmonic mean:**

x	12	5	8	4

Ans

x	$\frac{1}{x}$
12	0.0833
5	0.2
8	0.125
4	0.25
SUM	0.6583

$$\text{Harmonic Mean} = \text{H.M} = \frac{n}{\Sigma\left(\frac{1}{x}\right)}$$

$$= \frac{4}{0.6583}$$

$$= 6.0763$$

4. Write short answers to any SIX (6) questions: (12)

(i) **Define an angle.**

Ans An angle is defined as the union of two non-collinear rays with some common end points. The rays are called arms of the angle and the common end point is known as vertex of the angle.

(ii) **Convert $\frac{3\pi}{4}$ to degrees.**

Ans

$$\frac{3\pi}{4} = \frac{3\pi}{4} \times 1 \text{ radian}$$

$$= \frac{3\pi}{4} \times \frac{180^\circ}{\pi}$$

$$= 135^\circ$$

(iii) Define projection.

Ans The projection of a given point on a line is the foot of ⊥ drawn from the point on that line. However, the projection of given point P on a line AB is the point P itself.

(iv) Define circle.

Ans A circle is the locus of a moving point P in a plane, which is always equidistant from some fixed point O.

(v) Define secant.

Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vi) Define circumference of a circle.

Ans The boundary of a circle is called circumference. $2\pi r$ is the circumference of a circle with radius r.

(vii) Define sector of a circle.

Ans The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(viii) Define radius of a circle.

Ans The distance from the centre of the circle to any point on the circle is called radius of the circle.

(ix) Define circum circle.

Ans The circle passing through the vertices of triangle ABC is known as circum circle, its radius as circum radius and centre as circum centre.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$7x^2 + 2x - 1 = 0$

Ans As given

$$7x^2 + 2x - 1 = 0 \qquad \text{(i)}$$

$$7x^2 + 2x = 1$$

Dividing both sides by '7',

$$\frac{7x^2}{7} + \frac{2x}{7} = \frac{1}{7}$$

$$x^2 + \frac{2x}{7} = \frac{1}{7} \qquad \text{(ii)}$$

Adding both sides with $\left(\frac{1}{7}\right)^2$,

$$x^2 + \frac{2x}{7} + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2$$

$$(x + \frac{1}{7})^2 = \frac{1}{7} + \frac{1}{49}$$

$$(x + \frac{1}{7})^2 = \frac{7 + 1}{49}$$

$$(x + \frac{1}{7})^2 = \frac{8}{49}$$

By taking under root both sides,

$$\sqrt{(x + \frac{1}{7})^2} = \pm\sqrt{\frac{8}{49}}$$

$$x + \frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$x = \frac{-1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

Thus, solution set is $\left\{\frac{-1 \pm 2\sqrt{2}}{7}\right\}$.

(b) For what value of k, the expression $k^2x^2 + 2(k + 1)x + 4$ is perfect square. (4)

Ans Given, $k^2x^2 + 2(k + 1)x + 4$ (i)

Here, $a = k^2$, $b = 2(k + 1)$, $c = 4$

$$\begin{aligned}\text{Discriminant} &= b^2 - 4ac \\ &= \{2(k + 1)\}^2 - 4(k^2)(4) \\ &= 4(k^2 + 1 + 2k) - 16k^2 \\ &= 4k^2 + 4 + 8k - 16k^2 \\ &= -12k^2 + 8k + 4\end{aligned}$$

As expression (i) is a perfect square (given), so the roots must be rational and equal. Thus,

$$\text{Discriminant} = 0$$

$$-12k^2 + 8k + 4 = 0$$

$$12k^2 - 8k - 4 = 0$$

$$12k^2 - 12k + 4k - 4 = 0$$

$$12k(k-1) + 4(k-1) = 0$$

$$(k-1)(12k+4) = 0$$

$$k - 1 = 0 \quad ; \quad 12k + 4 = 0$$

$$k = 1 \quad ; \quad 12k = -4$$

$$; \quad k = \frac{-4}{12}$$

$$; \quad k = \frac{-1}{3}$$

Q.6.(a) If a : b = c : d (a, b, c, d ≠ 0) by using k-method show that $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$. **(4)**

Ans Given, $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$ (I)

And given ratio,

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

By letting, $\frac{a}{b} = k \quad ; \quad \frac{c}{d} = k$

$$a = bk \quad ; \quad c = dk$$

L.H.S of (I) $= \frac{a}{b}$

$$= \frac{bk}{b} \qquad \because \quad a = bk$$

$= k$ (II)

R.H.S of (I) $= \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$

$$= \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}} \qquad \therefore a = bk \text{ and } c = dk$$

$$= \sqrt{\frac{b^2k^2 + d^2k^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{k^2(b^2 + d^2)}{b^2 + d^2}}$$

$$= \sqrt{k^2}$$

$$= k \qquad \text{(III)}$$

From II and III, we get

L.H.S = R.H.S

Hence, $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$ Proved

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(b) Resolve into partial fraction: $\frac{9}{(x-1)(x+2)^2}$. (4)

Ans $$\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

By multiplying both sides with $(x-1)(x+2)^2$, we get

$$\frac{9}{(x-1)(x+2)^2}(x-1)(x+2)^2 = \frac{A}{(x-1)}(x-1)(x+2)^2 +$$

$$\frac{B}{(x+2)}(x-1)(x+2)^2 + \frac{C}{(x+2)^2}(x-1)(x+2)^2$$

$$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \qquad \text{(I)}$$

To find the value of 'A'; Put $x - 1 = 0$ in (I)

$$x - 1 = 0$$

$$x = 1$$

$$9 = A(1+2)^2 + B(1-1)(1+2) + C(1-1)$$

$$9 = A(3)^2 + B(0)(3) + C(0)$$

$$9 = A(9) + 0 + 0$$

$$\frac{9}{9} = A$$

$\Rightarrow$ $A = 1$

To find the value of 'C', put $(x+2)^2 = 0$ in (I)

$$(x+2)^2 = 0$$

$$x + 2 = 0$$
$$x = -2$$

$$9 = A(-2 + 2)^2 + B(-2 - 1)(-2 + 2) + C(-2 - 1)$$
$$9 = A(3)^2 + B(0)(3) + C(0)$$
$$9 = A(0)^2 + B(-3)(0) + C(-3)$$
$$9 = 0 + 0 - 3C$$
$$\frac{9}{-3} = C$$

$\Rightarrow$ $\boxed{C = -3}$

To find the value of 'B',

$$9 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1)$$
$$9 = A(x^2 + 4 + 4x) + B[x^2 + 2x - x - 2] + C(x - 1)$$
$$9 = Ax^2 + 4A + 4Ax + Bx^2 + Bx - 2B + Cx - C$$
$$9 = Ax^2 + Bx^2 + 4Ax + Bx + Cx + 4A - 2B - C \qquad \text{(II)}$$

By equating coefficients of x^2 on both sides, we get

$$0 = A + B$$
$$0 = 1 + B$$
$$-1 = B$$

$\Rightarrow$ $\boxed{B = -1}$

By putting the values of A, B and C in their relevant places, it is resolved that

$$\frac{9}{(x - 1)(x + 2)^2} = \frac{1}{x - 1} - \frac{1}{x + 2} - \frac{3}{(x + 2)^2}$$

Q.7.(a) If U = {1, 2, 3, 4, ----, 10}, A = {1, 3, 5, 7, 9}, B = {2, 3, 4, 5, 8}, then prove that (B – A)′ = B′ ∪ A. (4)

Ans L.H.S = (B – A)′

Firstly, B – A = {2, 3, 4, 5, 8} – {1, 3, 5, 7, 9}
= {2, 4, 8}

(B – A)′ = U – (B – A) = {1, 2, 3, 4, …, 10} – {2, 4, 8}
= {1, 3, 5, 6, 7, 9, 10}

R.H.S = B′∪A

Firstly, B′ = U – B = {1, 2, 3, 4, …, 10} – {2, 3, 4, 5, 8
= {1, 6, 7, 9, 10}

$B' \cup A = \{1, 6, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\}$
$= \{1, 3, 5, 6, 7, 9, 10\}$

So, L.H.S = R.H.S

(b) Find standard deviation 'S': **(4)**

9, 3, 8, 8, 9, 8, 9, 18

Ans

$$n = 8$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{9+3+8+8+9+8+9+18}{8}$$

$$\bar{x} = \frac{72}{8}$$

$$\bar{x} = 9$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
9	0	0
3	−6	36
8	−1	1
8	−1	1
9	0	0
8	−1	1
9	0	0
18	9	81
SUM	0	120

Standard Deviation:

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{120}{8}}$$

$$= \sqrt{15}$$

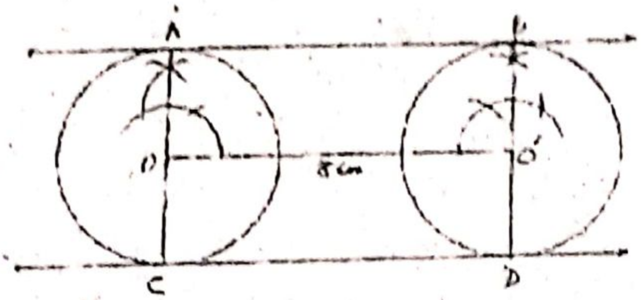
$$\boxed{S = 3.87}$$

Q.8.(a) Prove that: $\frac{1+\sin\theta}{1-\sin\theta}-\frac{1-\sin\theta}{1+\sin\theta}=4\tan\theta\sec\theta$.

Ans For Answer see Paper 2017 (Group-II), Q.8.(a).

(b) Two equal circles are at 8 cm apart. Draw two direct common tangents of this pair of circles. (4)

Ans



Step of Construction:

(i) Draw a line segment of 8 cm length.

(ii) Draw two circles of equal size on their centres O and O'.

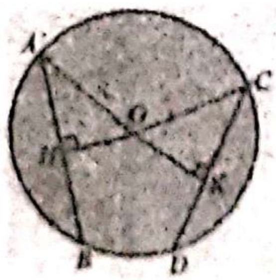
(iii) Take $\overline{OA}\perp\overline{OO'}$ and produce it towards O. Then, $\overline{OA}$ meets the circle at C.

(iv) Take $\overline{O'B}\perp\overline{OO'}$ and produce it towards O'. $\overline{O'B}$ meets the circle at D.

(v) Join A with B and C with D, and produce these both sides. Thus, AB and CD are the required common external tangents.

Q.9. Prove that two chords of a circle which are equidistant from the centre, are congruent.

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C.

So that we have $\angle rt\Delta^s$ OAH and OCK.

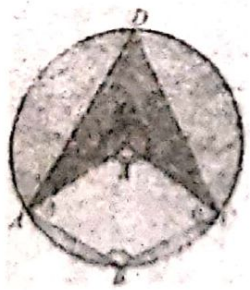
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly, $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle rt\ \Delta^s$ OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{AH} = \overline{CK}$	Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

OR

Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary.

Ans



Given:

ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:

$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$.

Write ∠1, ∠2, ∠3, ∠4, ∠5, and ∠6 as shown in the figure.

Statements	Reasons
Standing on the same arc ADC, ∠2 is a central angle whereas ∠B is the circumangle	Arc ADC of the circle with centre O.
$\therefore \quad m\angle B = \frac{1}{2}(m\angle 2)$ (i)	By theorem 1
Standing on the same arc ABC, ∠4 is a central angle whereas ∠D is the circumangle	Arc ABC of the circle with centre O.
$\therefore \quad m\angle D = \frac{1}{2}(m\angle 4)$ (ii)	By theorem 1
$\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2} m\angle 2 + \frac{1}{2} m\angle 4$ $= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle) *i.e.*, $m\angle B + m\angle D = \frac{1}{2}(4\angle rt)$ $= 2\angle rt$ Similarly, $m\angle A + m\angle C = 2\angle rt$	Adding (i) and (ii)